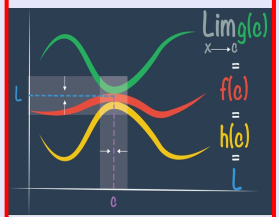
Calculus I

Lecture 4

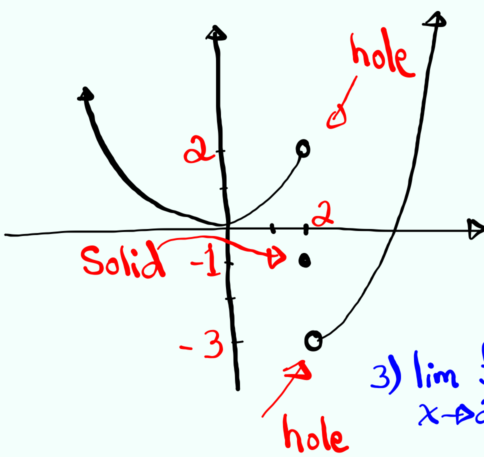


Feb 19-8:47 AM

Class QZ 3

Box Your Final Ans.

Consider the graph of f(x) below.



Find

1) $\lim_{x \to 2^+} f(x) = \boxed{-3}$

2) $\lim_{x \to 2^-} f(x) = \boxed{2}$

3) $\lim_{x \to 2} f(x)$ $\boxed{\text{DNE}}$

4) $f(2) = \boxed{-1}$

f(x) is not cont. at $x = 2$.

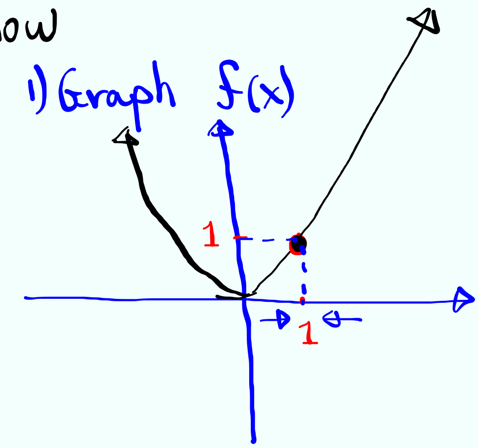
Jun 23-7:33 AM

Consider the function below

Piece-wise

$$f(x)=\begin{cases} x^2 & \text{if } x<1 \\ x & \text{if } x\geq 1 \end{cases}$$

1) Graph $f(x)$



2) Find $\lim\limits_{x\to 1} f(x) = \boxed{1}$

3) Find $f(1) = \boxed{1}$

4) Discuss Continuity of $f(x)$ at $x=1$.

$\lim\limits_{x\to 1} f(x) = f(1)$ therefore $\therefore$ cont. at $x=1$

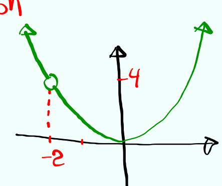
Jun 23-8:18 AM

$$f(x)=\frac{x^3+2x^2}{x+2}$$

1) Domain $x+2\neq 0$ $x\neq -2$ $(-\infty,-2)\cup(-2,\infty)$

2) Simplify $f(x)$, then graph

$f(x)=\dfrac{x^2\cancel{(x+2)}}{\cancel{x+2}} = x^2, x\neq -2$



3) Is $f(x)$ cont. at $x=-2$? Discuss.

$\lim\limits_{x\to -2^-} f(x)=4$

$\lim\limits_{x\to -2^+} f(x)=4$

$\Rightarrow \lim\limits_{x\to -2} f(x)=4$

$f(-2)$ undefined

There is a hole at $x=-2$

$f(x)$ is not Cont. at $x=-2$.

Jun 23-8:26 AM

Consider $f(x)$ below

$$f(x)=\begin{cases} x^2+x-K & \text{if } x<3 \\ Kx+7 & \text{if } x\geq 3 \end{cases}$$

1) $\lim\limits_{x\to 3^-} f(x) = 3^2+3-K = \boxed{12-K}$

2) $\lim\limits_{x\to 3^+} f(x) = K(3)+7 = \boxed{3K+7}$

3) Find K such that $\boxed{\lim\limits_{x\to 3} f(x) \text{ exist.}}$

$12-K=3K+7$

$12-7=3K+K$

$5=4K$

$\boxed{K=\frac{5}{4}}$

$f(x)$ will be Cont. at $x=3$ only if $K=\frac{5}{4}$

Jun 23-8:34 AM

$f(x)=2x^2-8x$

1) Evaluate $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0} \dfrac{2(x+h)^2-8(x+h)-2x^2+8x}{h}$

$=\lim\limits_{h\to 0} \dfrac{2(x^2+2xh+h^2)\cancel{-8x}-8h-2x^2\cancel{+8x}}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{2x^2}+4xh+2h^2-8h\cancel{-2x^2}}{h}$

$=\lim\limits_{h\to 0} \dfrac{\cancel{h}(4x+2h-8)}{\cancel{h}} = \lim\limits_{h\to 0}(4x+\cancel{2h}-8) = \boxed{4x-8}$

2) Solve $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = 0$

$4x-8=0$ $\quad\boxed{x=2}$

Jun 23-8:42 AM

$f(x) = \frac{x}{x-2}$

1) Find $\frac{f(x+h)-f(x)}{h}$, and Simplify

$\cancel{x^2} - \cancel{2x} + \cancel{hx} - 2h - \cancel{x^2} - \cancel{hx} + \cancel{2x}$

$$\frac{\frac{x+h}{x+h-2} - \frac{x}{x-2}}{h} = \frac{(x+h)(x-2) - x(x+h-2)}{h(x+h-2)(x-2)} = \frac{-2\cancel{h}}{\cancel{h}(x+h-2)(x-2)}$$

LCD $(x+h-2)(x-2)$

$$= \frac{-2}{(x+h-2)(x-2)}$$

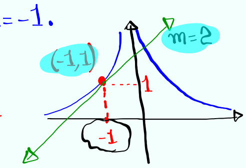
2) Evaluate $\lim_{h\to0} \frac{f(x+h)-f(x)}{h}$ at $x=3$.

$$\lim_{h\to0}\frac{-2}{(x+\cancelto{0}{h}-2)(x-2)} = \frac{-2}{(x-2)(x-2)} = \frac{-2}{(x-2)^2}$$

For $x=3$ $\frac{-2}{(3-2)^2} = \boxed{-2}$

Jun 23-8:53 AM

Find eqn of the tan. line on the graph of $f(x)=\frac{1}{x^2}$ at $x=-1$.



Hint

$$m_{\text{tan. line}} = \lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$m = \lim_{h\to0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h\to0} \frac{x^2 - (x+h)^2}{h(x+h)^2x^2}$$

LCD: $(x+h)^2x^2$

$$= \lim_{h\to0} \frac{\cancel{x^2} - \cancel{x^2} - 2xh - h^2}{h(x+h)^2x^2} = \lim_{h\to0} \frac{\cancel{h}(-2x-h)}{\cancel{h}(x+h)^2x^2}$$

$$= \lim_{h\to0} \frac{-2x - \cancelto{0}{h}}{(x+\cancelto{0}{h})^2\cdot x^2} = \frac{-2x}{x^2\cdot x^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

at $x=-1$

$m = \frac{-2}{(-1)^3} = \boxed{2}$

$y - y_1 = m(x-x_1)$

$y-1 = 2(x--1)$

$y-1 = 2x+2$

$\boxed{y=2x+3}$

Jun 23-9:06 AM

Prove $\lim_{x \to 4} (5x - 12) = 8$ ✓

$f(x) = 5x - 12$

$L = 8$

$a = 4$

1) Verify the limit. ✓

$\lim_{x \to 4} (5x-12) = 5(4) - 12$

$= 20 - 12 = \boxed{8}$ ✓

2) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|5x - 12 - 8| < \varepsilon$ 〃 $|x - 4| < \delta$

$|5x - 20| < \varepsilon$

$|5(x-4)| < \varepsilon$

$5|x-4| < \varepsilon \to |x-4| < \frac{\varepsilon}{5}$

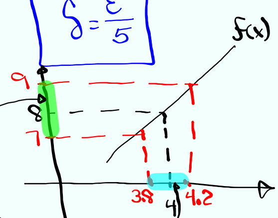
Pick $\boxed{\delta = \frac{\varepsilon}{5}}$

If $\varepsilon = 1 \to \delta = \frac{1}{5} = .2$

$x = 4.1$

$f(4.1) = 5(4.1) - 12$

$= 20.5 - 12 = \boxed{8.5}$



Jun 23-9:32 AM

Prove $\lim_{x \to -3} (-2x + 10) = 16$ ✓

$f(x) = -2x + 10$ $L = 16$ ✓ $a = -3$

Verify the limit. ✓

$\lim_{x \to -3} (-2x + 10) = -2(-3) + 10 = 6 + 10 = \boxed{16}$ ✓

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|-2x + 10 - 16| < \varepsilon$ 〃 $|x - (-3)| < \delta$

$|-2x - 6| < \varepsilon$ 〃 $|x + 3| < \delta$

$|-2(x+3)| < \varepsilon$

$2|x+3| < \varepsilon \to |x+3| < \frac{\varepsilon}{2}$

Pick $\boxed{\delta = \frac{\varepsilon}{2}}$

Jun 23-9:40 AM

For $\boxed{\varepsilon = .1}$, find $\delta > 0$ such that $\lim_{x\to 6} x^2 = 36$.

$f(x) = x^2$

$L = 36$ verify the limit.

$a = 6$ $\lim_{x\to 6} x^2 = 6^2 = 36$

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$|x^2 - 36| < \varepsilon$ " $|x-6| < \delta$

$|(x+6)(x-6)| < \varepsilon$ " $|x-6| < \delta$

$\underbrace{|x+6|}_{\text{Bound}}\ \underbrace{|x-6|}_{\text{Keep}} < \varepsilon$

So if $|x+6| < C$, then $|x-6| < \frac{\varepsilon}{C}$

For now $\delta = \frac{\varepsilon}{C}$

It is common for $\delta < 1$
(for now)

$|x-6| < 1$

+6 $-1 < x-6 < 1$

+6 $5 < x < 7$

$11 < x+6 < 13$ → $|x+6| < 13$

$C = 13$

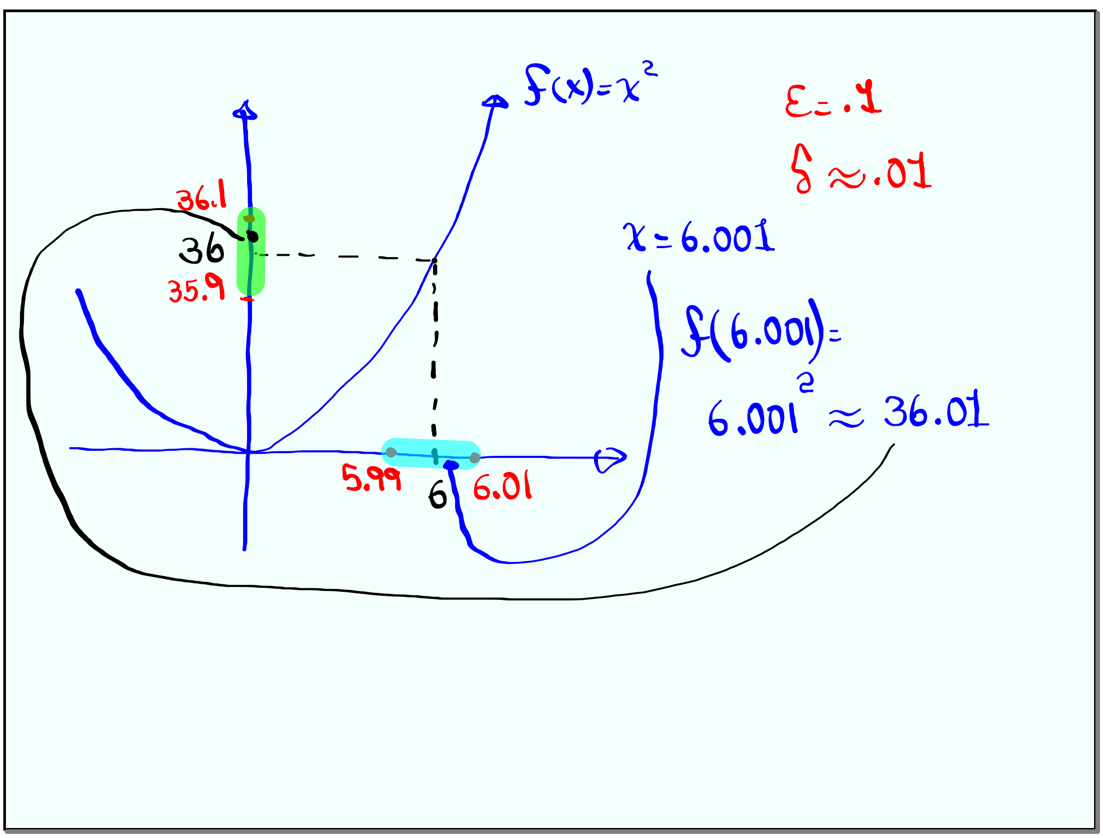
$\delta = \min\left\{1, \frac{\varepsilon}{13}\right\}$

for $\varepsilon = .1$ → $\delta = \min\left\{1, \frac{.1}{13}\right\}$

$= \min\left\{1, \frac{1}{130}\right\}$

$\delta = \frac{1}{130} \approx .01$

Jun 23-9:46 AM



Jun 23-9:57 AM

For $\boxed{\varepsilon=1}$, find $\delta>0$ such that $\lim_{x\to2}(x^2+2x)=8$.

$f(x)=x^2+2x$ $\quad L=8 \quad a=2$

$\lim_{x\to2} f(x)=\lim_{x\to2}(x^2+2x)=2^2+2(2)=4+4=8$ ✓

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$|x^2+2x-8|<\varepsilon$ " $|x-2|<\delta$

$|(x+4)(x-2)|<\varepsilon$ " $|x-2|<\delta$

$\underbrace{|x+4|}_{\text{Bound}}\,\underbrace{|x-2|}_{\text{Keep}}<\varepsilon$ $\quad \delta=\frac{\varepsilon}{c}$

If $|x+4|<c$, then $|x-2|<\frac{\varepsilon}{c}$

For $\delta<1$ $\quad |x-2|<1$

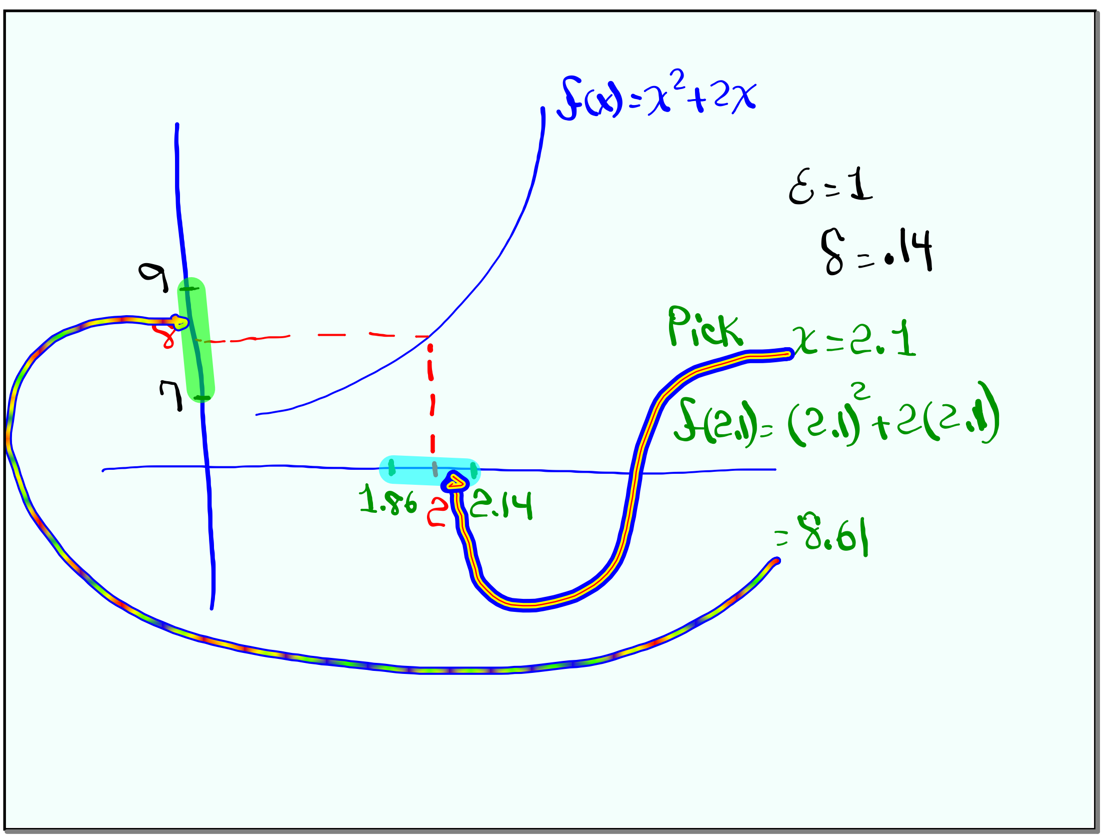
$+6$ $\quad -1<x-2<1$

$5<x+4<7$ $\quad$ So $|x+4|<7$

For $\varepsilon=1$ $\quad c=7$

$\delta=\min\{1,\frac{1}{7}\}=\frac{1}{7}\approx .14$ $\quad \delta=\min\{1,\frac{\varepsilon}{7}\}$

Jun 23-10:00 AM



Jun 23-10:10 AM

For $\varepsilon > 0$, find a $\delta > 0$ such that $\lim_{x \to \frac{1}{2}} \frac{1}{x} = 2$. ✓

$f(x) = \frac{1}{x}$, $L = 2$, $a = \frac{1}{2}$

Verify the limit ✓

$\lim_{x \to 1/2} \frac{1}{x} = \frac{1}{1/2} = 1 \div \frac{1}{2} = 1 \cdot \frac{2}{1} = 2$ ✓

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{1}{x} - 2\right| < \varepsilon$ 〃 $\left|x - \frac{1}{2}\right| < \delta$

$\left|\frac{1 - 2x}{x}\right| < \varepsilon$ 〃 $\left|x - \frac{1}{2}\right| < \delta$

$\left|\frac{-2x + 1}{x}\right| < \varepsilon$

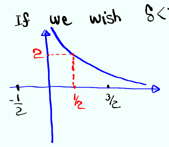
$\left|\frac{-2\left(x - \frac{1}{2}\right)}{x}\right| < \varepsilon$ For now $\delta = \frac{\varepsilon}{C}$

Bound Keep

$\frac{2}{|x|}\left|x - \frac{1}{2}\right| < \varepsilon$

If $\frac{2}{|x|} < C$, then $\left|x - \frac{1}{2}\right| < \frac{\varepsilon}{C}$

If we wish $\delta < 1$, $\left|x - \frac{1}{2}\right| < 1$



$-1 < x - \frac{1}{2} < 1$

Add $\frac{1}{2}$

$-\frac{1}{2} < x < \frac{3}{2}$

we have a Problem we need to Pick δ to be less than $1/2$, maybe $1/4$, $1/5$, $\frac{1}{10}$.

Jun 23-10:14 AM

Introduction to derivative:

For function $f(x)$, its first derivative is $f'(x)$ and is given by

"f prime of x"

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

only when limit exists.

Jun 23-10:31 AM

find $f'(x)$ for $f(x) = x^3$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{- x^3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} = \lim_{h \to 0}(3x^2 + \cancel{3xh}^{\,0} + \cancel{h^2}^{\,0})$$

$$= \boxed{3x^2}$$

$f(x) = x^3$

$f'(x) = 3x^2$

Jun 23-10:34 AM

find $f'(x)$ for $f(x) = x^2 - 8x + 5$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 8(x+h) + 5 - x^2 + 8x - 5}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{- 8x} - 8h \cancel{+ 5} \cancel{- x^2} \cancel{+ 8x} \cancel{- 5}}{h}$$

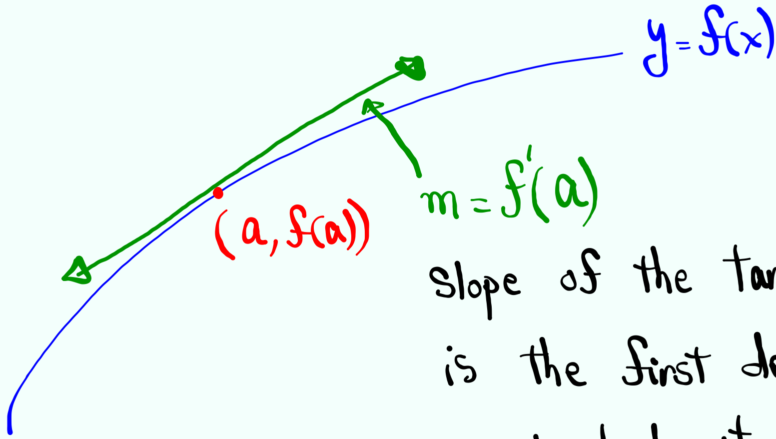
$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 8)}{\cancel{h}} = \lim_{h \to 0}(2x + \cancel{h}^{\,0} - 8)$$

$f(x) = x^2 - 8x + 5$

$f'(x) = 2x - 8$

$$= \boxed{2x - 8}$$

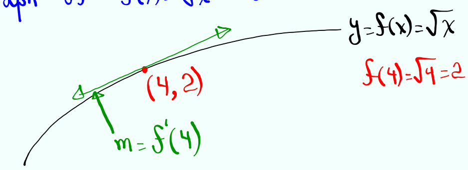
Jun 23-10:39 AM



Slope of the tan. line is the first derivative evaluated at tangent pt.

Jun 23-10:46 AM

Find slope of the tan. line to the graph of $f(x) = \sqrt{x}$ at $x = 4$.



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{\cancel{x} + \overset{1}{\cancel{h}} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x + \overset{\to 0}{h}} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$$m = f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2} = \boxed{\frac{1}{4}}$$

for $x = 9$

$$m = \frac{1}{2\sqrt{9}} = \frac{1}{6}$$

Jun 23-10:49 AM

Find $f'(x)$ for $f(x) = \frac{1}{x}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\boxed{\frac{1}{x+h}} - \boxed{\frac{1}{x}}}{h} = \lim_{h \to 0} \frac{x - (x+h)}{h(x+h)\cdot x}$$

LCD: $(x+h)x$

$$= \lim_{h \to 0} \frac{\cancel{x} - \cancel{x} - \cancel{h}^{-1}}{\cancel{h}(x+h)\cdot x} = \lim_{h \to 0} \frac{-1}{(x+\cancel{h}^{0})\cdot x} = \frac{-1}{x \cdot x}$$

$$= \boxed{\frac{-1}{x^2}}$$

Jun 23-10:57 AM